Reynolds Number Worksheet

The **Reynolds Number (Re)** is a dimensionless number that can be used to predict flow patterns in different fluid flow situations, such as air over an aircraft wing or liquid in a pipe. Reynolds number calculations are often used to design products or experiments. For instance, if you were testing the efficiency of a small-scale model of a car in a wind tunnel you would design your experiment so the full scale and model have similar Reynolds Numbers.

For flow in a tube or pipe, the Reynolds Number can be used to predict whether the flow will be **laminar** or **turbulent**.

Definitions

dimensionless number: a number representing a physical property that has no unit scale

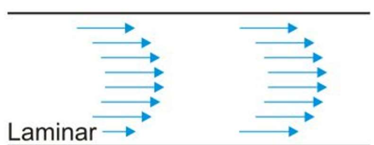
Reynolds Number*:* the ratio of inertial forces to viscous forces

turbulent flow*:* flow in which the fluid undergoes irregular fluctuations

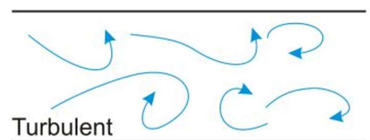
laminar flow*:* flow in which the fluid moves in smooth paths or layers

Stokes Flow*:* fluid flow where inertial forces are small compared with viscous forces

Re < 2000

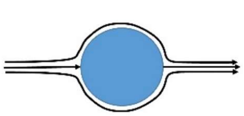


Re > 4000

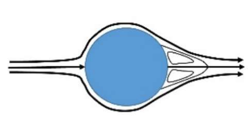


In microfluidics, the Reynolds Number is typically very low, often much less than 1, and this is a kind of extreme laminar flow known as **Stokes Flow**. In Stokes Flow, fluids will stay laminar even when flowing around corners or obstacles. This can affect things like heat transfer and reaction rates, as flow through a microfluidic channel typically only mixes through diffusion.

$\mathrm{Re} \approx 0.01$



$\mathrm{Re} \approx 20$



$\mathrm{Re} \approx 2{,}000$

The Reynolds number depends on the fluid density, velocity and viscosity as well as the dimensions of the channel. Since the Reynolds Number is a **dimensionless number,** all units (m, kg, etc.) will cancel out for the final value.

$$Re = \frac{\rho v d}{\mu}$$

ρ = density

v = velocity

d = diameter of channel

μ = dynamic viscosity

For water at ~20 C

ρ_{water} = 1000 kg/m3

μ_{water} = 1.0 N-s/m2 = 1.0 kg/m-s (1 N = 1 kg-m/s2)

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Example: Will water flowing through a 1 cm diameter circular pipe at 10 cm^3/s be laminar or turbulent?

What we know:

$$density\ of\ water:\ \ \rho_{water} = 1000\ \mathrm{kg/m^3}$$

$$dynamic\ viscosity\ of\ water:\ \ \mu_{water} = 1\,\frac{\mathrm{kg}}{\mathrm{m{\cdot}s}}$$

$$flow\ rate\ in\ the\ channel:\ \ F = 10\ \mathrm{cm^3/s}$$

$$channel\ diameter:\ \ d = 1\ \mathrm{cm}$$

$$Re = \frac{\rho v d}{\mu}$$

You have all needed values to plug into the Reynolds Number equation except the velocity (*v*). You know the flow rate (volume/time), but you need to know the velocity (distance/time).

What's the difference between flow rate and velocity?
flow rate is the *volume* of fluid that moves past a surface per unit time
velocity (or speed) is the *distance* the fluid moves per time.

To calculate the velocity from the flow rate, use the equation:

$$velocity = \frac{Flow\ rate}{Cross\ sectional\ area}$$

Step 1: Find the velocity in m/s:

$$v = \frac{F}{A}$$

Where *F* = flow rate and *A* = area.

$$A_{circle} = \pi r^2 = \pi(\tfrac{d}{2})^2 = \pi(\tfrac{1\ cm}{2})^2 = 0.78\ cm^2$$

$$v = \frac{F}{A} = \frac{10\ \frac{cm^3}{s}}{0.78\ cm^2} = 12.8\ \frac{\cancel{cm}}{s} \times \frac{1\ m}{100\ \cancel{cm}} = \mathbf{0.128\ \frac{m}{s}}$$

Step 2: Convert the diameter into meters:

$$d = 1\ \mathrm{cm} \times \frac{1\ m}{100\ cm} = 0.01\ m$$

Step 3: Plug into the Reynolds Number equation:

$$\mathrm{Re} = \frac{\rho v d}{\mu} = \frac{(1000\ \frac{\mathrm{kg}}{\mathrm{m^3}})(0.128\ \frac{\mathrm{m}}{\mathrm{s}})(0.01\mathrm{m})}{(1.0\ \frac{\mathrm{kg}}{\mathrm{m\ s}})}$$

Step 4: Make sure all of your dimensions cancel out and solve for Re.

$$\mathrm{Re} = 1.28\frac{(\cancel{\frac{\mathrm{kg}}{\mathrm{m^3}}})(\cancel{\frac{\mathrm{m}}{\mathrm{s}}})(\cancel{\mathrm{m}})}{(\cancel{\frac{\mathrm{kg}}{\mathrm{m\ s}}})}$$

$$\mathbf{Re = 1.28}$$

This flow will be laminar.

Problem 1: You have water flowing in a 2.5 mm diameter circular channel at a flow rate of 10 µl/s.

a. What is the fluid velocity in m/s?

Hint: 1 µl = 1 mm^3

b. What is the Reynolds Number?

$$Re = \frac{\rho v d}{\mu}$$

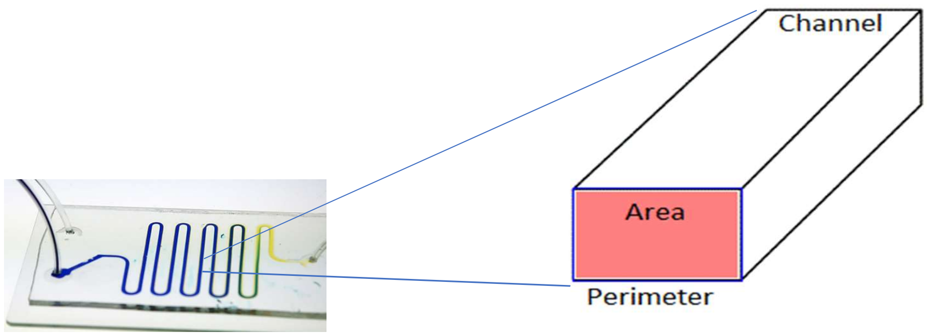
Remember:

ρ_{water} = 1000 kg/m^3
μ_{water} = 1.0 N-s/m^2 = 1.0 kg/m-s

The Reynolds Number equation assumes a circular cross section in the channel. You can calculate the *hydraulic diameter* of a non-circular chip with the equation:

$$d_H = \frac{4A}{P}$$

$$= \frac{4 \times (Cross\ sectional\ area)}{Perimeter}$$



Problem 2: The main channel in the Y-microfluidic chip included in the Basic Microfluidic Education Kit has a rectangular cross-section with a width of 0.4 mm and depth of 0.1 mm.

a. What is the channel hydraulic diameter?

b. If the flow rate across the chip is 100 µl/min, what is the velocity in meters per second?

Remember: 1 µl = 1 mm^3

c. Calculate the Reynolds Number for the channel, assuming water flow.
Don't forget, the Reynolds Number is a dimensionless number, so all units will cancel.

d. With this Reynolds Number, would you expect the fluids from the two channels to mix? Explain.

Reynolds Number Worksheet - Solutions

Problem 1: You have water flowing in a 2.5 mm diameter circular channel at a flow rate of 10 µl/s.

a: What is the fluid velocity in m/s?

$$v = \frac{F}{A}$$

$$F = volume\ flow\ rate = 10\ \mu\text{l/s} = 10\ \text{mm}^3\text{/s}$$

$$A = area = \pi r^2 = \pi(\frac{d}{2})^2 = \pi(\frac{2.5}{2})^2 = 4.9\ \text{mm}^2$$

$$v = \frac{F}{A} = \frac{10\ \frac{mm^3}{s}}{4.9\ mm^2} = 2.04\ \frac{\cancel{mm}}{s} \times \frac{1\ m}{1000\ \cancel{mm}} = \mathbf{0.002\ \frac{m}{s}}$$

b. What is the Reynolds Number?

$$\text{Re} = \frac{\rho v d}{\mu} = \frac{(1000\ \frac{\text{kg}}{\text{m}^3})(0.002\ \frac{\text{m}}{\text{s}})(2.5\ \text{mm})}{(1.0\ \frac{\text{kg}}{\text{m s}})}$$

$$\text{Re} = \frac{(\cancel{1000}\ \frac{\cancel{\text{kg}}}{\cancel{\text{m}^3}})(0.002\ \frac{\cancel{\text{m}}}{\cancel{\text{s}}})(2.5\ \cancel{\text{mm}})(\frac{1\ \cancel{\text{m}}}{\cancel{1000\ \text{mm}}})}{(1.0\ \frac{\cancel{\text{kg}}}{\cancel{\text{m s}}})}$$

Re = 0.005

Problem 2: The main channel in the Y-microfluidic chip included in the Basic Microfluidic Education Kit has a rectangular cross-section with a width of 0.4 mm and depth of 0.1 mm.

a: What is its hydraulic diameter?

$$d_H = \frac{4A}{P}$$

$$A = cross\ sectional\ area = w \times D = 0.4\,mm \times 0.1\,mm = 0.04\ \text{mm}^2$$

$$P = perimeter = 2 \times (w + D) = 2(0.4\,mm + 0.1\,mm) = 1\ \text{mm}$$

$$d_H = \frac{4A}{P} = \frac{4 \times 0.04\ \text{mm}^2}{1\ \text{mm}} = \mathbf{0.16\ mm}$$

b: If the flow rate on the chip is 100 μl/min, what is the velocity in meters per second?

$$v = \frac{F}{A}$$

$$F = volume\ flow\ rate = 100\ \mu\text{l/min} = 100\ \text{mm}^3\text{/min}$$

$$A = cross\ sectional\ area = 0.04\ \text{mm}^2$$

$$v = \frac{F}{A} = \frac{100\ \frac{mm^3}{min}}{0.04\ mm^2} = 2500\ \frac{\cancel{mm}}{\cancel{min}} \times \frac{1\ \cancel{min}}{60\ s} \times \frac{1\ m}{1000\ \cancel{mm}} = \mathbf{0.041\ \frac{m}{s}}$$

c: Calculate the Reynolds Number for the channel, assuming water flow.

$$\text{Re} = \frac{\rho v d_H}{\mu} = \frac{\left(1000\ \frac{\text{kg}}{\text{m}^3}\right)\left(0.041\ \frac{\text{m}}{\text{s}}\right)(0.16\ \text{mm})}{\left(1.0\ \frac{\text{kg}}{\text{m s}}\right)}$$

$$\text{Re} = \frac{(\cancel{1000}\ \frac{\cancel{\text{kg}}}{\cancel{\text{m}^3}})(0.0041\ \frac{\cancel{\text{m}}}{\text{s}})(0.16\ \cancel{\text{mm}})(\frac{1\ \cancel{\text{m}}}{\cancel{1000\ \text{mm}}})}{(1.0\ \frac{\cancel{\text{kg}}}{\cancel{\text{m s}}})}$$

Re = 0.00066